High Dimensional Electromagnetic Design Problems Based on Sequential Subspace Optimization Method and Grey Correlation Analysis

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Abstract —We present a sequential subspace optimization method (SSOM) to deal with the high dimensional electromagnetic design problems with grey correlation analysis (GCA) method. To implement the proposed method, we firstly use GCA method to divide the initial design space into two subspaces, significant factors space and non-significant factors space. Then we sequentially optimize those two spaces, and we can get the optimal solutions. Furthermore, we present two optimization strategies for SSOM, direct optimization and approximate model optimization. From a design example of cylindrical voice coil actuator, we can see that SSOM can produce satisfactory solutions and the cost of finite element analysis can be reduced remarkably.

I. INTRODUCTION

Optimizing an electromagnetic device usually includes two steps. One is the model construction for the device, and the other is the model optimization. For the former step, finite element model (FEM) is the most widely used model. It is accurate, but it is always computationally expensive or time consuming to carry out, especially for high dimensional design problems. To reduce the cost of finite element analysis, magnetic circuit model and several approximate models, such as response surface model and Kriging model have been introduced as the surrogate models [1], [2]. They are proved fast, but not very accurate, especially for the high dimensional problems. For example, the needed FEM sample points for a seven variables problem with five level full factorial design is $5^7 = 78125$, and this cost is too expensive and impractical in many cases.

In our former work, we have introduced sequential optimization method (SOM) to solve such problems [3]. SOM is composed of several sequential model optimization processes and it can significantly reduce the computation cost. However, the cost for optimizing a high dimensional problem is still expensive as an additional sampling process of algorithm optimization is needed in the dimension reduction process.

For a high dimensional problem, there are many variables, and some of them have significant effects to the objectives, while the others do not. If we separate each of them, a lot of FEM costs can be saved. For example, if there are only 2 significant variables in the above example,

the needed FEM sample points maybe $3⁵*5²=6075$, which is less than 10% of the former sample. Therefore, it is essential to develop a set of new methods for high dimensional design problems with the consideration of the significance of the factors.

Fig. 1. Flowchart of SSOM

II. SSOM

Fig. 1 illustrates the flowchart of SSOM. There are mainly five steps.

1) Define the optimization problems, including design variables, objectives, and constraints.

2) Sample initial data with FEM and implement the GCA method. From GCA method, we can divide the total space (X) into two subspaces, significant factors space (X1) and non-significant factors space (X2). GCA method is a sensitivity and significance analysis method which is conducted to evaluate the importance of the design factors [4]. The traditional sensitivity analysis, such as analysis of variance, is a ''changing one factor at a time method'', namely, one factor is varied while all the others are fixed. However, the simplicity of this method may yield unreliable results and an inadequate conclusion because factors are uncertain. Moreover, there are many coupling relations in the variables of high dimensional problems and some points needed in the analysis of variance may be

impossible for practical application. The basic concept of GCA is to determine if a relationship among a series of data is close, based on the degree of similarity among the geometric shapes of the data series curves. It is a better choice for many engineering problems, so we present in this paper.

3) Optimize the significant factors space X1. In the implementation, the parameters in X2 are set to be fixed values which come from the analysis solutions of GCA. Two optimization models will be employed, FEM and Kriging approximate model [1], [2].

4) Optimize the significant factors space X2. In the implementation, the parameters in X1 are set to be constant which come from the optimization of the last step.

5) Terminate step. If $\Delta f / f \leq \varepsilon$, output the optimal solutions. Otherwise, go to step 3 and implement the optimization process again.

III. CYLINDRICAL VOICE COIL ACTUATOR

Fig. 2 illustrates an axisymmetrical model of cylindrical voice coil actuator (CVCA) [5], [6]. There are seven design parameters for this problem. The optimization objective and constraints are listed as follows.

Min:
$$
f =
$$
 Mass of total device
\n
$$
\begin{cases}\ng_1: \text{Coil mass} = 10g \\
g_2: F = 5N, \text{ when } D=4.25 \text{mm} \\
g_3: \overline{B}_i \le 1T \text{ in area } 1,2,3 \text{ in Fig.3} \\
g_4: s_{\min} + s_{\max} + x_6 \le x_3 \\
g_5: x_3 \le x_1 + x_6 - s_{\min}\n\end{cases}
$$

where s_{min} , s_{max} are constant, and the design ranges and some material parameters can be seen in [5], [6].

Fig. 2. Design model of CVCA: (a) material, (b) parameters

IV. DISCUSSIONS AND RESULTS

Table I lists the optimization solutions for CVCA with the proposed method. Three solutions are shown in the table. One is from the direct optimization with FEM and differential evolution algorithm (DEA column); the second is from SSOM with FEM (FEM column); and the last is from SSOM with Kriging model (Kriging column).

Fig. 3. Magnetic relevant part in the third constraint

TABLE I OPTIMAIZATION RESULTSS OF CVCA

Parameter	Unit	DEA	FEM	Kriging
x_1	mm	9.8682	10.7522	11.8667
\mathcal{X}_{2}	mm	8.9773	8.7813	8.6751
x_3	mm	13.4858	14.8021	15.1090
x_4	mm	1.7226	1.6148	1.6022
x_{5}	mm	3.1425	3.3231	3.5822
x ₆	mm	4.4563	6.0477	6.2694
x_7	mm	2.5688	2.3946	2.5582
Force	N	5.0127	4.9906	4.9050
Coil mass	g	9.9413	9.9908	10.0098
Total mass	g	79.1953	84.0488	91.0728
FESP		6725	4316	2267

From the table, we can see that SSOM can produce satisfactory solutions. Compared with solution from DEA, the objectives from two SSOM are a little bigger, and the solution from Kriging is about 15.0% more than that from DEA. But for the cost of finite element analysis, the direct optimization is the biggest, and for the SSOM with Kriging model, the finite element sample points (FESP) has been saved by 66.29% compared with that of DEA. Therefore, the proposed method is very promising.

V. REFERENCES

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